Why Are Earthquakes So Gentle?

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* should not be blamed if this fantastic story is really fantasy

Summary (Stress Paradox)

- Seismic data indicates that the effective shear stress accelerating the sides of a fault is less than 1.5 MPa for large earthquakes.
- Frictional sliding stress cannot exceed 3 MPa for 3 m slips, or the fault melts.
- High rupture velocities indicate that fracture energies do not exceed the radiated seismic energy.
- Therefore, the spatially averaged shear stress is less than 10 MPa for large earthquakes.
- Since there are earthquakes at 10 MPa, the "strength" of the crust is only 10 MPa.
- Laboratory measurements (much smaller length scale) indicate that the yield strength on faults is 200 MPa.
- Changes in shear stress can approach 200 MPa at some points along a rupture.
- Therefore the "strength" of the crust is 200 MPa (Paradox).

Summary
(Size Matters)

- Slip is spatially heterogeneous in earthquakes and stress in the crust is strongly heterogeneous.
- Friction changes dramatically from high static friction to very low sliding friction during rupture (at least for large events).
- The strength of the crust is described by the spatial power spectrum of stress in the crust.
- Strength (stress averaged over the rupture) becomes a statistical parameter that depends on the size of the rupture.
- Large earthquakes are so gentle because the Earth is so large.
- I don’t much about small earthquakes, which may be very different.

Follow the Energy in Earthquakes

- $\Delta W = \text{change in potential energy}$

$$= \iint_S \left(\sigma_0(x,y) - \frac{\Delta\sigma(x,y)}{2} \right) D(x,y)\, dS$$

$$\sigma_0 \equiv \text{Initial shear stress}$$

$$\Delta\sigma \equiv \text{Change in shear stress from quake}$$

$$D \equiv \text{earthquake slip}$$

$$S \equiv \text{rupture surface}$$

- From seismology/geodesy we know $D(x,y)$
- From which we can calculate $\Delta\sigma(x,y)$.
- If we can estimate ΔW, then we can estimate σ_0.

Earthquake Energy Balance

$$\Delta W = E_F + E_G + E_R$$

$E_F \equiv$ frictional sliding work on fault plane

$E_G \equiv$ inelastic work off the fault

(plasticity at crack tip)

$E_R \equiv$ radiated wave energy

By estimating $E_F, E_G,$ and $E_R,$ we can estimate σ_0.

A Simple-Minded model of an Earthquake Rupture

1. Consider a 50-km long by 20-km deep vertical strike-slip rupture.
2. Assume that fault begins slip with friction of $0.6 \cdot P(z)$.
3. Significant melting occurs after several cm of slip.
4. Friction stress becomes viscous flow problem ...
5. Fault shear tractions drop to zero and stress drop is total.
6. An average stress drop of 100 MPa would imply an average slip of 80 m and particle velocities of 35 km/hr (or 10 m/s).
7. Even if you could construct a building to survive the motion, you would be killed by a wall running into you at 35 km/hr!!

How much energy can go into sliding friction?

- Assume that friction stress, σ_F, at depth, z, is given by coefficient of friction, μ_F (□ 0.6), and confining pressure P

$$\sigma_F \approx P(z)\mu_F$$

- Frictional heat per unit of rutpure area is

$$\Delta H = \sigma_F D$$

- Melting a 1-cm wide zone of rock requires about 15 MJ/m^2
- Pressure at 10km is 300 MPa (3,000 bars or 50,000 psi)
- 10 cm of slip at 10 km and $\mu_F = 0.6$ melts a 1-cm-wide zone!

  10 m of slip melts a 1m wide zone!

  (pointed out by Paul Richards in 1976)

Fracture Energy

$$E_G = SG_0$$

- $G_0 \approx 100 \ {}^{\text{J}}\!/_{\text{m}^2}$ for tensile fracture of polycrystalline materials.
- But we need to find $300 \ {}^{\text{MJ}}\!/_{\text{m}^2}$ if $\bar{\sigma}_0 \approx 100$ MPa.
- If $G_0 = 0$, then cracks run at their limiting speed, V_L.
  $V_L = .92\beta$ for mode II and $V_L = \beta$ for mode III.
- Radiated energy scales as particle velocity squared so

$$E_R \approx \left(\frac{V_R}{V_L}\right)^2 (\Delta W - E_F) = \left(\frac{V_R}{V_L}\right)^2 (E_R + E_G)$$

or

$$E_G \approx \left(\frac{V_L^2}{V_R^2} - 1\right) E_R$$

- Since $V_R \approx 0.9V_L$, E_G cannot be much larger than E_R !!

Seismological data implies that average shear stress is low in large earthquakes

- $\Delta W = E_R + E_G + E_F$
- $E_R \approx 1.5\bar{D}S$ MJ/m^3 (from seismology)
- For mode III steady-state rupture with $V_R = 0.8\beta$

$$E_G \approx \tfrac{1}{2} E_R$$

- If $\Delta T < 1{,}000^o$ K in a 1 cm wide zone then

$$E_F < 16\, S \ \text{MJ/m}^2$$

- For our 20 km x 50 km fault with 3 m slip,

$$\sigma_0 = \frac{\Delta W}{SD} + \Delta\sigma/2$$, and thus $\sigma_0 < 10$ MPa.

The Seismological answer seems to be that we could not survive large earthquakes if the shear stress is high.

What does a low E_R / M_0 mean?

$$\frac{E_R}{M_0} = \frac{\left(\sigma_0 - \Delta\sigma/2 - \bar{\sigma}_F\right)}{\mu}\left(1 - \sqrt{\frac{\beta - V_R}{\beta + V_R}}\right) \quad \text{(mode III)}$$

- So a low value could indicate either high average sliding friction $\bar{\sigma}_{\mathrm{F}}$, or low rupture velocity for smaller earthquakes.
- Seismic observations indicate that

$$\mathrm{M}_0 = \mu S \bar{D} \propto T_R^3 \approx \left(\frac{\sqrt{S}}{V_R}\right)^3$$

and since $\Delta\sigma \propto \dfrac{\mu \bar{D}}{\sqrt{S}}$, then $\Delta\sigma \propto \dfrac{1}{V_R^3}$

- V_R is usually assumed to be independent of moment and the observation is usually interpretted that $\Delta\sigma$ is independent of moment.
- If V_R increases with earthquake size, then $\Delta\sigma$ decreases with earthquake size.
- There are examples of 100 MPa stress drops for M 3 earthquakes, but there are not for large earthquakes (praise God).

But isn’t shear stress (strength) in the crust high???

- Laboratory derived coefficient of friction implies shear of 100-200 MPa at 10 km.
- Borehole stress measurements seem compatible with high stress.
- How can we support mountains if shear stress is less than 10 MPa?
- Slip is very heterogeneous on faults and implies local stress differences of 100 MPa.
- Can these observations be compatible with the low average stresses in large earthquakes?

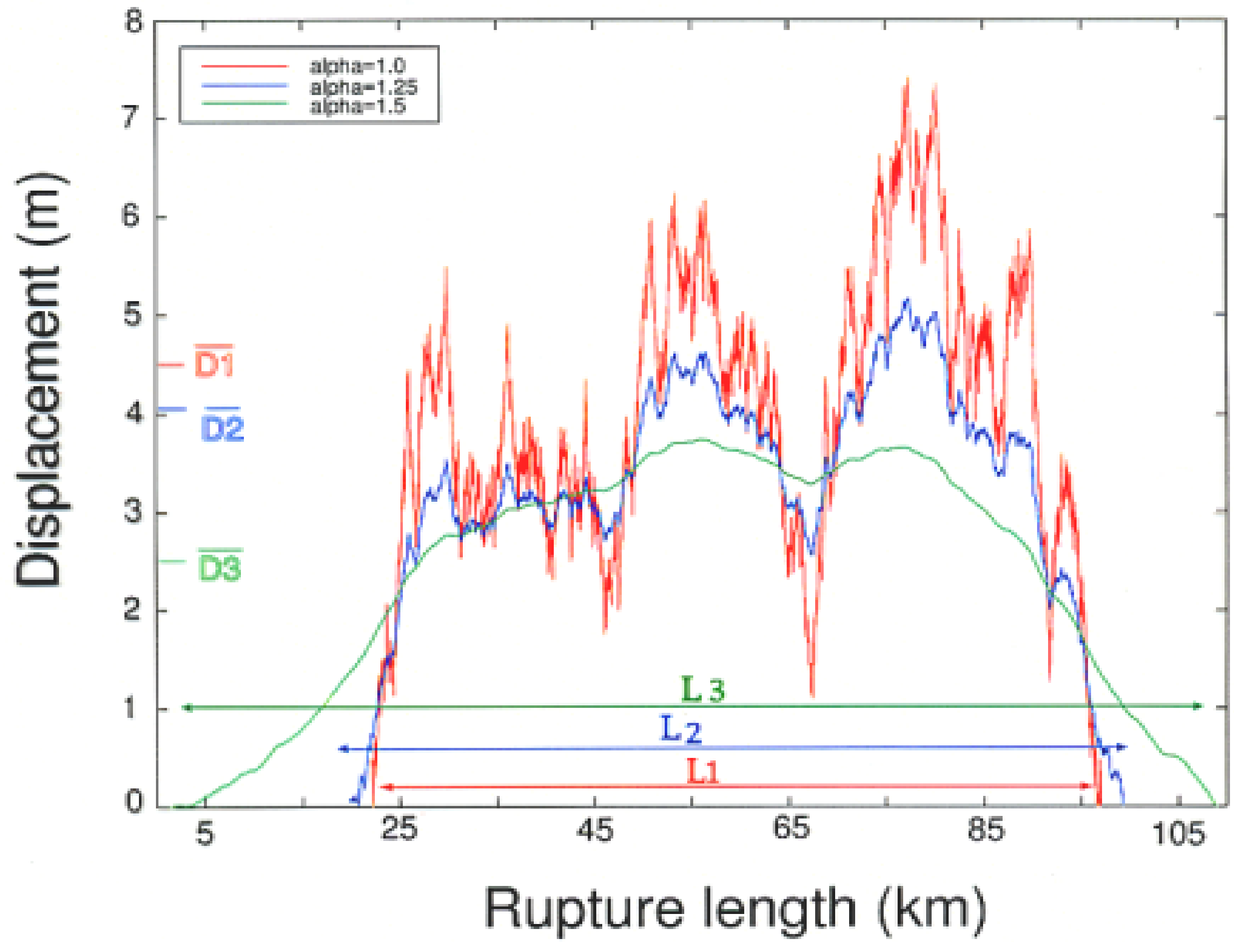
Spatially Heterogeneous Slip on faults is a Key Clue!

- Jing Liu and Chris Dicaprio are working on this problem.
- Consider a slip model in which slip is a stochastic, stationary function of location on the fault.
- Assume that an individual earthquake consist of a spatially contiguous patch.
- Statistical properties of the slip determine:

  1) (average slip)/(rupture dimension)

  2) Frequency/Magnitude statistics

Given equal surface areas, islands with rougher topography
have higher average elevations



$$\tilde{D}(k) \propto k^{-\alpha} \text{ , where } k \equiv \text{ wavenumber}$$

From J. Liu and T. Heaton

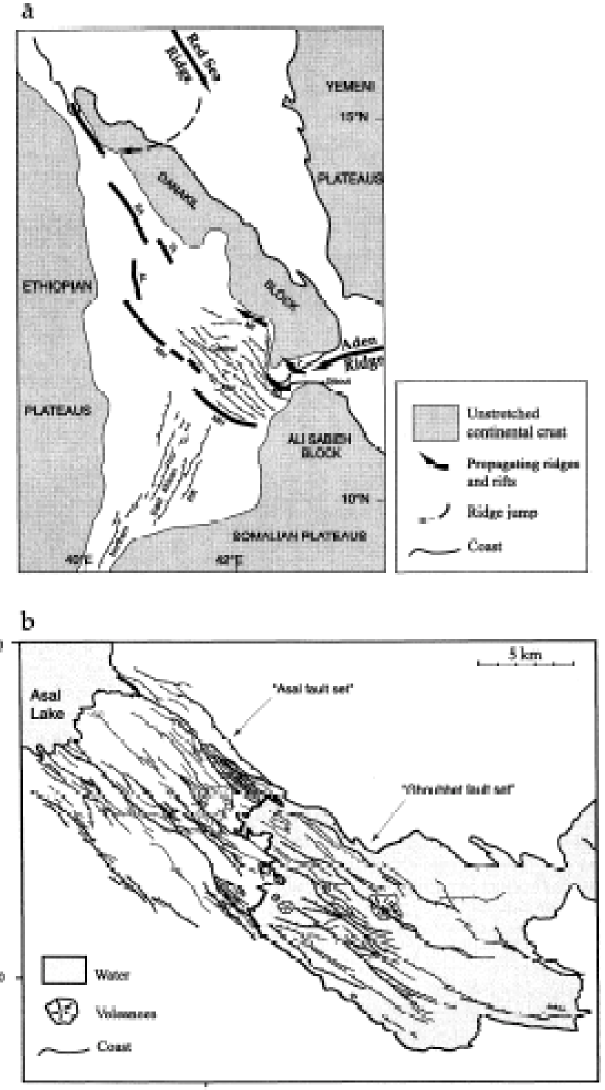
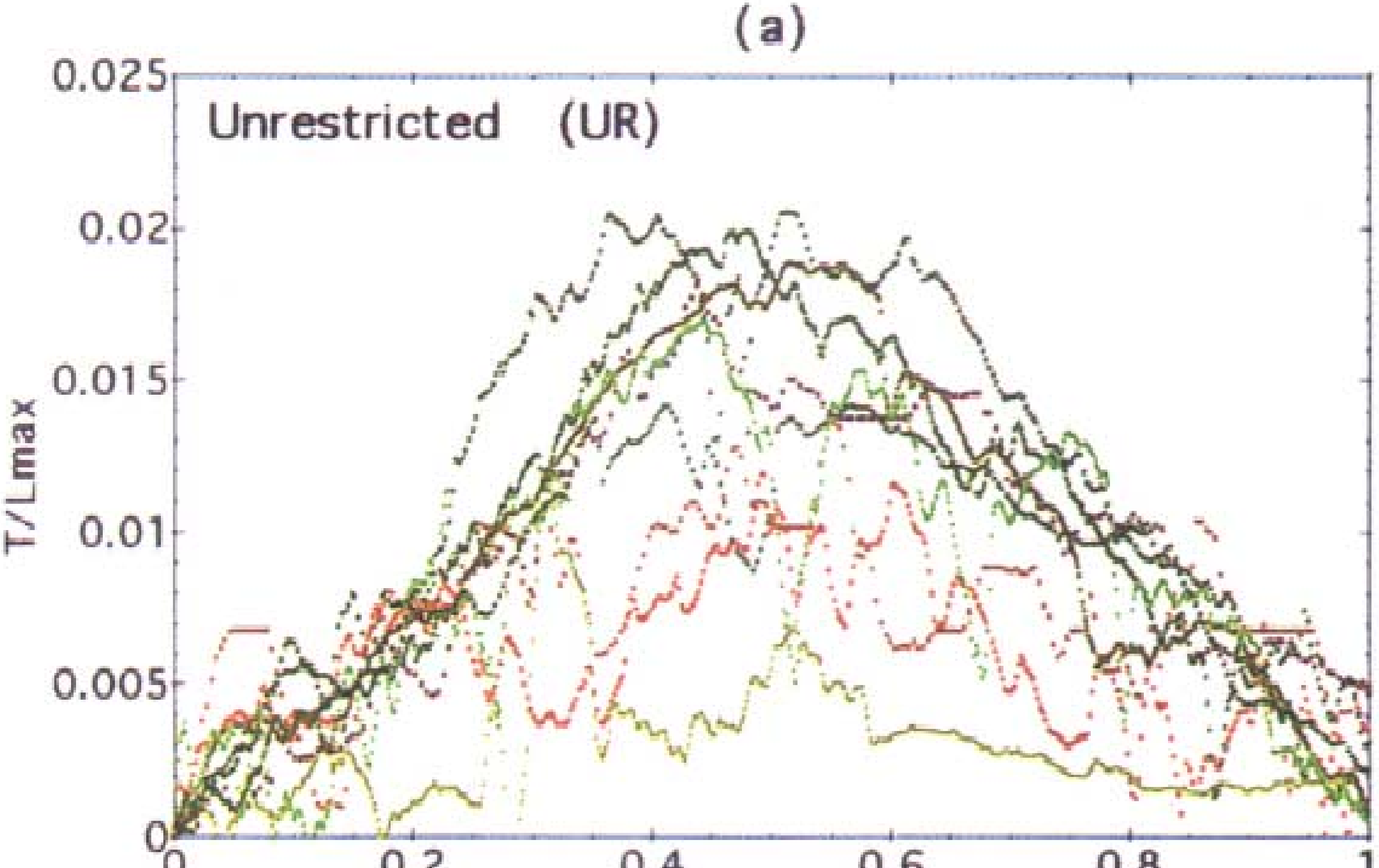


Figure 1. (a) Tectonic setting of Afar depression [from Manighetti et al., 1998]. About 20 Ma, the Red Sea ridge jumped into Afar and propagated southeastward through development of distinct rifts (EA, Erta'Ale; TA, Tat'Ali; AL, Alayta; MH, Manda Hararo; see *Manighetti et al.* [1997, 1998] for details). More recently (last 2 Myr), the Aden ridge entered and propagated westward into Afar, forming another series of rifts that are presently growing and propagating northwestward, as indicated by arrows (T, AG, and MI for Tadjoura, Asal-Ghoubbet, and Manda Inakir rifts, respectively). Between the two rifting zones (i.e., in central Afar), deformation is distributed on a dense network of NW striking normal faults. From these we selected most of the "Afar faults" (the few other ones are taken in northern East African Rift). In both central Afar and East African Rift, only major faults are represented. Box indicates location of Figure 1b, where "Asal" and "Ghoubbet" faults were analyzed. (b) Map of Asal-Ghoubbet faults (modified from *Audin et al.* [2001]). Most of the faults are examined in the present paper.



(a)
Unrestricted (UR)
T/Lmax
0.025
0.02
0.015
0.01
0.005
0
0
0.2
0.4
0.6
0.8
1

What does heterogeneous slip tell us about stress in the Crust?

$$\sigma(x) = \sum_{\text{history}} \Delta\sigma(x) + \sigma_{\text{tectonic}}$$

- σ_{tectonic} is approximately uniform since it is applied at a distance
- Change in stress is linear sum of spatial derivatives of displacement w.r.t. distance, or on the fault

$$\Delta\sigma(\mathrm{x}) \propto \mu \frac{\partial \mathrm{D}}{\partial \mathrm{x}}$$

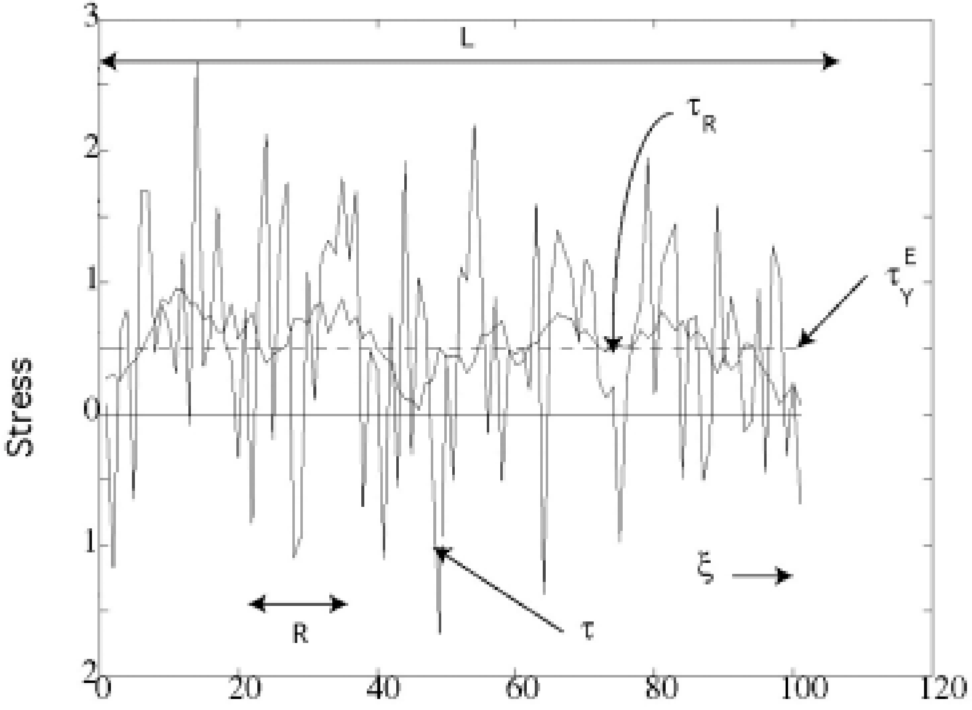
- Therefore, if $\tilde{D}(k) \propto k^{-\alpha}$, then $\Delta\tilde{\sigma}(k) \propto k^{-\alpha+1}$.

$$\tilde{\sigma}(k) = \sum_{\text{history}} \Delta\tilde{\sigma}(k) + \sigma_{\text{tectonic}}\delta(k)$$

$$= \sigma_{\text{internal}} k^{-\gamma} + \sigma_{\text{tectonic}}\delta(k)$$

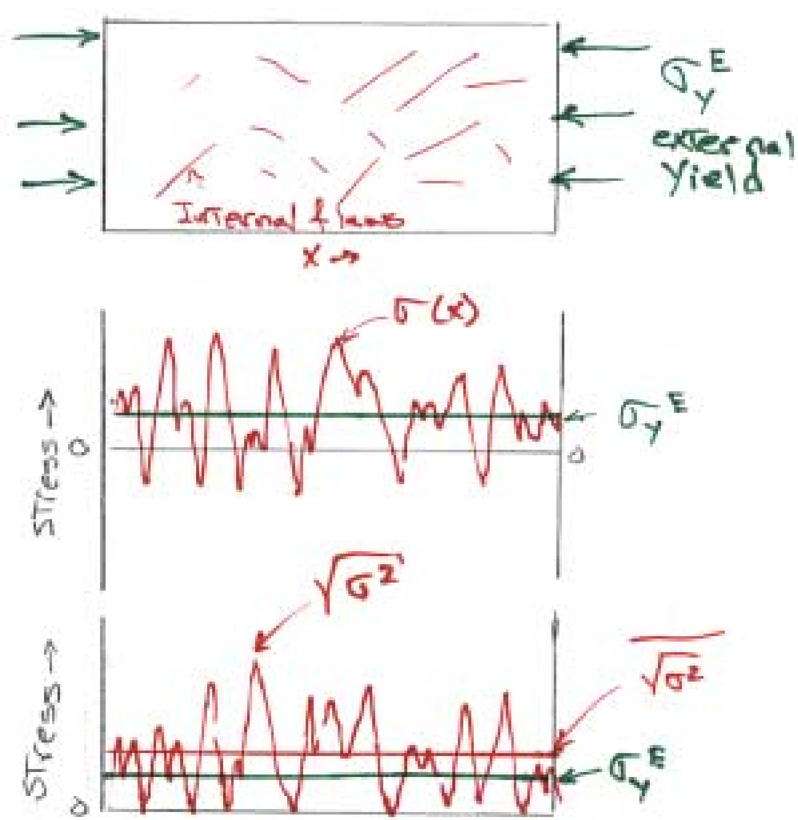
You cannot physically measure heterogeneous stress at a point

- Stress measurements are always averages over some length scale R.
- In the case of earthquakes, this is the dimension of the rupture, R.
- In the Earth, the fact that slip is so heterogeneous, and that average stress is so small means that stress must actually be both positive and negative when viewed at short length scales.



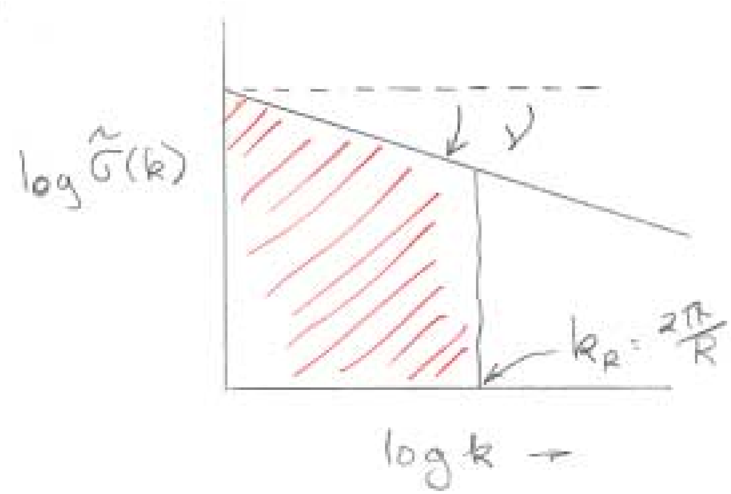
How can you define “strength” when stress is heterogeneous?

- In the lab, the external yield stress is measured = stress averaged over the entire sample.
- In the Earth, we estimate the size of the stress in the Earth and call it the strength.
- Strength must be a positive quantity.
- Seismologist’s “strength” estimates are similar to taking the spatial average of the root-mean-square of the stress.



Power-Law Critical Stress

- Remember that we found that Fourier spectrum of stress may obey a power law.
- If the material fails (through time) in such a way that the internal stress at *all* values of *R* is comparable to the external yield stress at that dimension, then the material is in a critical state at all length scales and the stress can be described with a power law.
- If the stress is critical, then there is no inherent length scale.
- The amplitude of "rms strength" is determined by the integral under the power spectrum out to the wavenumber with R.



What makes the stress so heterogeneous in the first place?

- Remember the problem that the friction during sliding is low (very low).
- The static friction must be high to sustain the large slip heterogeneity.
- There must be a dramatic change in friction between static and dynamic.

Particle-velocity-dependent friction produces heterogeneous slip?

- Friction depends on slip velocity, but slip velocity depends on friction (highly nonlinear positive feedback system).
- The slip at any point depends on the distance between the rupture front and the healing front. Both velocities are unsteady, producing highly heterogeneous slip as a function of space (Cochard and Madariaga).
- This nonlinear rupture freezes in short scale heterogeneities (large small-scale stresses).

Conclusions

- Sliding friction is very low in large earthquakes.
- Average stress is low for large earthquakes.
- Stress is strongly heterogeneous.
- Geophysicists mean stress averaged over a length scale when they say strength. Strength decreases with length scale.
- Average stress, fracture energy, and friction may all be higher for small earthquakes.
- We don’t yet know enough about size scaling of average stress, fracture energy and friction to understand the size scaling of effective stress.